LECTURE II:

☞ : ORIGIN OF THE SEESAW SCALE AND IMPLICATIONS FOR UNIFICATION

Why Seesaw is theoretically so appealing?

☞ **Adding N_R to std model makes fermion spectrum quark-lepton symmetric.**

☞ **Makes the spectrum also left-right symmetric: under Parity**

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \leftrightarrow \begin{pmatrix} u_R \\ d_R \end{pmatrix}; \quad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \leftrightarrow \begin{pmatrix} N_R \\ e_R \end{pmatrix};$$

Expanding of weak gauge symmetry

☞ **Standard model:** $\partial^\mu J_\mu = 0$
but $Tr(B-L)$ **and** $Tr(B-L)^3$ **both** $\neq 0$**;**

Add RH neutrino to SM and both $Tr(B-L)$ **and** $Tr(B-L)^3$ **both** $= 0$**;**

This means $(B-L)$ **is a gauge symmetry !!**

and the electroweak gauge group expands to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$**; i.e. the left-right symmetric model of weak interactions.**

Some details of left-right symmetric models:

☞ Details

➢ Gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

➢ Matter: $SU(2)_L$ Doublets: $Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$; $\psi_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$;

$SU(2)_R$ doublets: $Q_R \equiv \begin{pmatrix} u_R \\ d_R \end{pmatrix}$;$\psi_R \equiv \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$

Higgs: $\phi(2,2,0) \equiv \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$;

$\Delta_R(1,3,+2) \equiv \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$

➢ $\mathcal{L}_Y = h_u \bar{Q}_L \phi Q_R + h_d \bar{Q}_L \tilde{\phi} Q_R + h_e \bar{\psi}_L \tilde{\phi} \psi_R + h.c.$
$+f(\psi_R \psi_R \Delta_R + L \leftrightarrow R)$

Fermion masses

☞ **Masses arise from symmetry breaking**

➢ $<\phi^0>=\begin{pmatrix}\kappa & 0\\ 0 & \kappa'\end{pmatrix}$ and $<\Delta_R^0>=v_R$

➢ $<\phi>$ gives masses to quarks and charged leptons only

➢ $m_\nu \neq 0$ arises from the seesaw matrix (coming up).

Features of left-right symmetric models

☞ **other features**

1. weak interactions become parity conserving
   $\mathcal{L}_{wk} = \frac{g}{2\sqrt{2}}(\vec{W}_{\mu,L} \cdot \vec{J}^{\mu}_{L} + \vec{W}_{\mu,R} \cdot \vec{J}^{\mu}_{R})$
2. Electric charge: $Q = I_{3L} + I_{3R} + \frac{B-L}{2}$
   Involves all physical quantum numbers
3. This generalization of Gell-Mann-Nishijima formula to weak interactions implies that:
   $\Delta I_{3R} = -\frac{B-L}{2}$ i.e. Neutrino is a Majorana mass purely because of group theory and its mass is linked to parity violation !!

R. N. M., Pati; Senjanovic, (1974-75)

Neutrino mass linked to parity violation

☞ Questions for Left-right models

1. Why are low energy weak int. V-A ?
2. Why $m_\nu \ll m_{u,d,e}$?
3. How high is the Parity breaking scale ?
4. How to experimentally test the idea ?

Both questions have the same answer:

☞ **BREAK PARITY AT SCALE MUCH ABOVE THE W_L MASS**

➢ $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow G_{std} \rightarrow U(1)_{em}$

$M_{\nu,N} = \begin{pmatrix} 0 & 0 \\ 0 & M_R \end{pmatrix} \rightarrow \begin{pmatrix} fv_L & h_\nu v \\ h_\nu^T v & fv_R \end{pmatrix}$ (type II SEESAW)

➢ As before, $m_\nu \simeq fv_L - \frac{h_\nu^2 v^2}{fv_R}$; $(v_L \sim \frac{v_{wk}^2}{v_R})$
Strength of V+A currents $\propto \frac{1}{v_R^2}$;
as the scale of parity violation $v_R \rightarrow \infty$, $m_\nu \rightarrow 0$;

➢ SMALLNESS OF m_ν CONNECTED TO THE SUPPRESSION OF V+A currents.

➢ Existence of B-L symmetry and left-right symmetry is the first thing we learn from ν-mass discovery !

Implication of Parity for seesaw

☞ **New contribution to seesaw :**

$m_\nu \simeq f\frac{v_{wk}^2}{v_R} - \frac{h_\nu^2 v_{wk}^2}{f v_R}$; **(Type II seesaw)**

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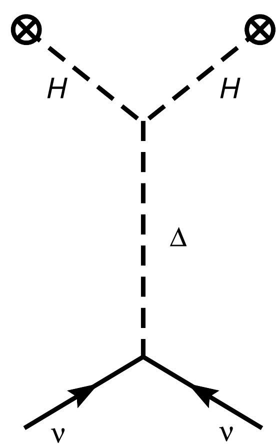

Figure 11: **Feynman diagram for type II seesaw**

Parity $\rightarrow$ Type II seesaw

TeV seesaw scale in LR models

☞ **The type I seesaw formula is given by:**

$$\mathcal{M}_\nu \;=\; -M_D^T M_R^{-1} M_D \sim -\frac{h_\nu^2 v_{wk}^2}{f v_R}$$

Expt: $m_{\nu_3} \sim 0.05$ **eV; if** $v_R \sim$ **TeV, means** $h_\nu \sim 10^{-6}$**;**
(Compare this with electron Yukawa coupling in the standard model $(= 3 \times 10^{-6}$**)- not very different !!)**

It is quite OK to have TeV scale seesaw.

New W_R **and** Z' **bosons in the TeV range; can be explored at LHC (see later)**

☞ : NEUTRINO MASS AND GRAND UNIFICATION

Basic idea of grand unification

Pati,Salam; Georgi, Glashow; Georgi, Quinn, Weinberg

☞

• All forces unify at some high scale;

• All matter i.e. quarks and leptons unify at that scale:

• A new symmetry (local) symmetry of physics that embodies the SM symmetry emerges at that scale and above.

GRAND UNIFICATION

☞ Why Grand unify ?

- ➢ Unification of quarks and leptons provides hope for understanding lepton masses in terms of quark masses- a first step towards solving the flavor puzzle ?
- ➢ It explains electric charge quantization.

Standdard model does not grand unifiy !

☞ **Gauge coupling runnung is determined by the low energy theory.**
For SM, the equations are:

$$\frac{d\alpha_i^{-1}}{dt} = \frac{b_i}{2\pi}$$

with $b_1 = -\frac{4}{3}N_g - \frac{1}{3}T_H$; $b_{2,3} = \frac{11N}{3} - \frac{4}{3}N_g - \frac{1}{3}T_H$ **with** $N = 2, 3$

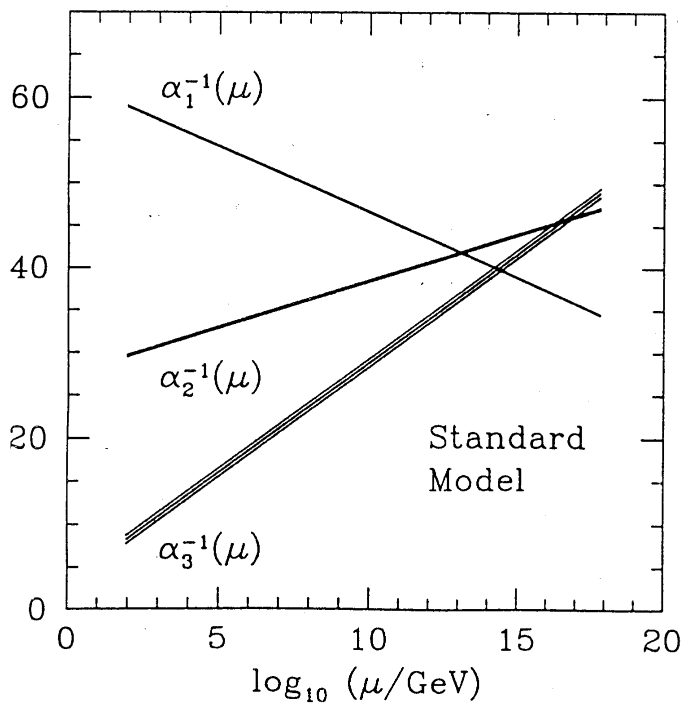

Figure 12: Coupling unification in supersymmetric theories

Note that $SU(2)$ and $SU(3)$ couplings meet at 10^{16} GeV but not $U(1)$; Supersymmetry cures that.

Example of grand unification

☞ **SUSY at TeV scale and no new physics until GUT scale;**

$b_1 = -\frac{33}{5}$; $b_{2,3} = \ (3N - 2N_g - T_H)$; **all couplings meet nicely.**

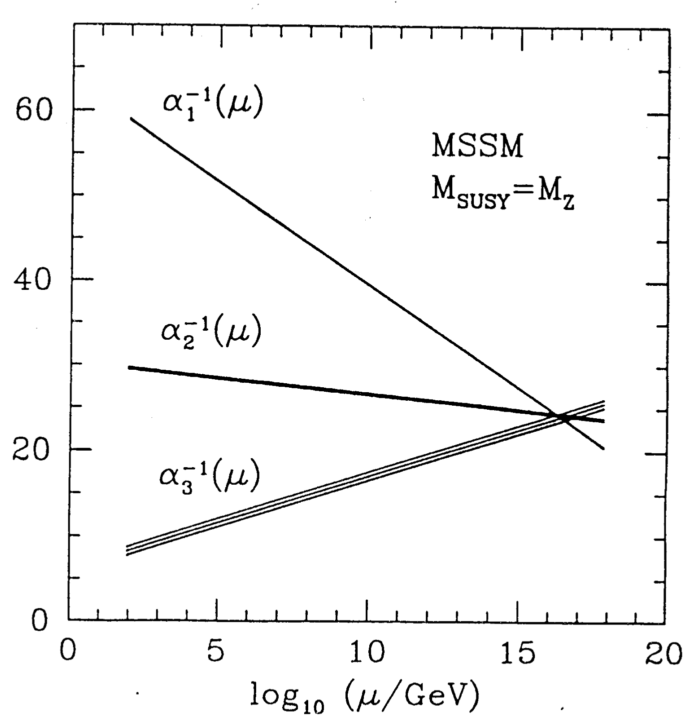

Figure 13: Coupling unification in supersymmetric theories

$M_U \simeq 2 \times 10^{16}$ **GeV;**

No SUSY but simple GUT

☞ **No SUSY and no new physics till SEESAW scale and the LR Symmetry till GUT**
(B) $G_{STD} \rightarrow SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c \rightarrow$ **(with no supersymmetry) and** $M_{B-L} \sim 10^{14}$ **GeV (the seesaw scale).**

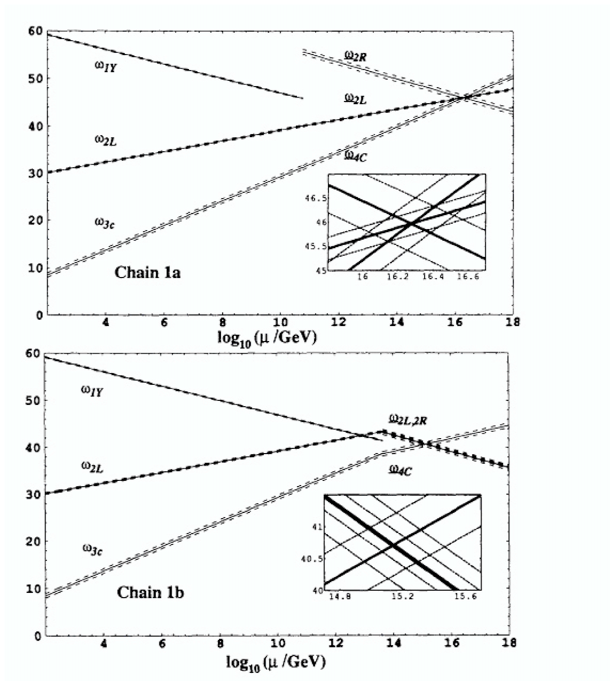

Figure 14: Coupling unification in non-supersymmetric left-right theories with int. seesaw scale

Requirement of coupling unification predicts seesaw scale around 10^9-10^{13} GeV

Proton decay: Key test of grand unification

☞ **Example of simple SU(5) model of Georgi and Glashow:**

Fermions: $\bar{5} = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e^- \end{pmatrix}$ **and** $10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & u_3 \\ & & 0 & u_3 & d_3 \\ & & & & e^+ \\ & & & & 0 \end{pmatrix}$

Quarks and leptons in the same multiplet and therefore gauge bosons connect quarks and leptons as well as quarks to anti-quarks leading to proton decay.

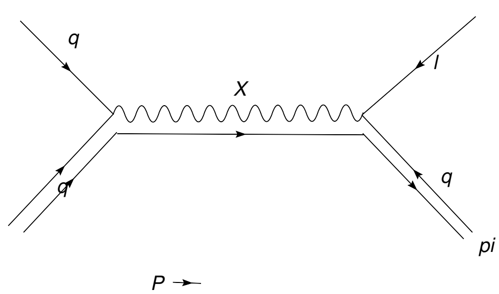

Figure 15: Proton decay in GUT theories- Gauge boson exchange

☞ **: Present in all simple GUT theories: fairly model independent except for the overall unification scale.**

$A(p \rightarrow e^+\pi^0) \sim \frac{g_U^2}{M_U^2}$

Prediction: $\tau_{p\rightarrow e^+\pi^0} \sim 10^{36\pm1}\left(\frac{M_U}{2\times10^{16}\ GeV}\right)^4$ **yrs**

Present lower limit: $\tau_{p\rightarrow e^+\pi^0} \geq 5\times10^{33}$ **yrs.**

Some of Super-K, IMB3,Soudan2 data on other proton decay modes

☞

mode	Lower limit in 10^{32} yrs
$p \longrightarrow e^+ + \pi^0$	**50**
$p \longrightarrow \bar{\nu} K^+$	**23**
$n \longrightarrow \bar{\nu} + K^0$	**1.3**
$p \longrightarrow \mu^+ + K^0$	**13**
$p \longrightarrow e^+ + K^0$	**10**
$p \longrightarrow \mu^+ \pi^0$	**43**
$p \longrightarrow \gamma e^+$	**98**
$p \longrightarrow \gamma \mu^+$	**82**
$n \longrightarrow e^+ \pi^-$	**1**

SUSY SU(5) model

☞ The simplest GUT model (circa 1980s)

- Fermions: $\bar{\mathbf{5}} = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e^- \end{pmatrix}$ and $\mathbf{10} = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & u_3 \\ & & 0 & u_3 & d_3 \\ & & & & e^+ \\ & & & & 0 \end{pmatrix}$
- : Higgs $\mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{24}$.
- Predicts: at M_U, $m_b = m_\tau$; very good prediction

  Also predicts $m_s = m_\mu$; $m_d = m_e$; VERY BAD PREDICTION!!
- No explanation of neutrino mass:
- Proton decay problem:

Proton decay in SUSY SU(5)

☞ **New graphs contribute to proton decay in GUT theories due to the existence of superpartners**

Non-susy theories: P-decay operator: $QQQL/M_U^2$ **whereas SUSY theories, it is** $QQ\tilde{Q}\tilde{L}_s/M_U$**;**
Note the change in power dependence !!

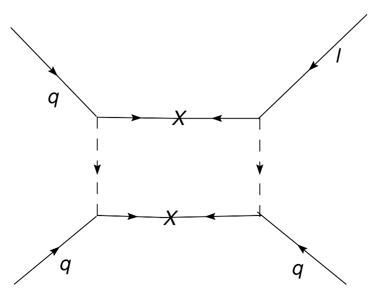

Figure 16: Dominant Diagram for proton decay in supersymmetric GUT theories

☞ **Decay mode** $p \to \bar{\nu} M^+$**;**
Life time: $(\tau_P)^{-1} \simeq [\frac{f^2}{M_U M S}]^2 \left(\frac{\alpha}{4\pi}\right)^2 m_p^5$ **Implies**
$\tau_{p\to K^+\bar{\nu}} \leq (10^{32})^{-1}$ **yrs**

Expt: $\tau_{p\to K^+\bar{\nu}} \geq 3 \times 10^{33}$ **yrs.**

Possible to cure it by giving up predictivity !!

m_ν and Grand unification

☞

$M_U \simeq 2 \times 10^{16}$ GeV; not far from
$M_{seesaw} \sim 2 \times 10^{14}$ GeV

$$M_R \simeq M_U$$

☞

➢ raises the hope that seesaw scale and GUT scale are same;

➢ Perhaps neutrino masses and mixings can be predicted due to higher symmetry of GUT theories which will reduce number of free parameters;

SO(10) SUSY GUT and neutrinos

Georgi; Fritzsch and Minkowski, 75

☞ **unification of all 16 fermions of one generation**

➢ $\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$ into **16** dim. rep of SO(10)

➢ Contains the N_R needed for seesaw automatically

➢ Contains the B-L subgroup which broken appropriately, gives R-parity as a natural symmetry and hence a stable dark matter

➢ None of these properties hold for SU(5)

Some useful group theory for SO(10)

☞ **SO(10) is almost like Lorentz group which is SO(3,1);**

Just like Lorentz group has spinor representation which is 4-dimensional and splits into two chiral 2-comp representations, SO(10) has a spinor rep which is 16-dim.

The general formula for the dim of of spinor rep of SO(2N) is 2^{N-1} dimensional.

Like there are Lorentz vectors and tensors, SO(10) has vector and tensor reps:
$H_\mu = \mathbf{10}$; $S_{\mu\nu} = \mathbf{54}$ **dim (sym);** $A_{\mu\nu} = \mathbf{45}$ **(anti-sym.);**
$\Sigma_{\mu\nu\lambda} = \mathbf{120}$ **(anti-sym);** $\Delta_{\mu\nu\lambda\sigma\tau} = \mathbf{126}$ **etc.**

$\mathbf{16} \otimes \mathbf{16} \;=\; \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126}$ **helps to write Yukawa couplings that are SO(10) invariant.**

Spinor of SO(10) for fermions of SM

☞

$$\begin{pmatrix} u \\ u \\ u \\ d \\ d \\ d \\ \nu \\ e \\ u^c \\ u^c \\ u^c \\ d^c \\ d^c \\ d^c \\ \nu^c \\ e^c \end{pmatrix}$$

Breaking SO(10) down

☞

- (i) SO(10)$\longrightarrow$ SU(5)$\longrightarrow$ std model or
- (ii) SO(10)$\longrightarrow$ $SU(2)_L \times SU(2)_R \times SU(4)_c \longrightarrow$ std model
- In either case one must break B-L symmetry

Predictions from one SO(10) model

☞

- ➢ Use only **126** to break B-L

- ➢ Yukawa coupling
$$\mathcal{L}_Y = h_{ab}\psi_a\psi_b H + f_{ab}\psi_a\psi_b\overline{\Delta}$$

- ➢ two pairs of Higgs doublets: one (H_u, H_d) from H and another from $\overline{\Delta}$

- ➢ Counting of parameters- 3 from h,6 from f plus 4 vevs minus $M_Z \rightarrow$ total of 12 parameters; vrs

  10 (quarks)+3 (e,$\mu + \tau$) + 18 for seesaw in the standard model (total of 31)

Predicting neutrino masses and mixings in minimal SO(10) with 126

☞ **CP conserving case as an example**

➢ Input: masses of (e, μ, τ), six quarks plus three CKM angles;

➢ All parameters of the fermion sector are determined; hence all (but one) neutrino masses and mixing angles predicted

☞

➢ Equations for fermion mass matrices

$$M_u = h < H_u > + f < \Delta_u >$$
$$M_d = h < H_d > + f < \Delta_d >$$
$$M_e = h < H_d > - 3f < \Delta_d >$$
$$M_{\nu^D} = h < H_u > - 3f < \Delta_u >$$

➢ It follows that

$$f \;=\; \frac{1}{4 < \Delta_d >}(M_d - M_\ell)$$

(Relation valid at GUT scale)

Large mixing from type II seesaw

☞ Triplet vev dominance in seesaw

- ➢ Seesaw formula in SO(10)
  $\mathcal{M}_\nu \simeq f\frac{v_{wk}^2}{v_R} - M_{\nu^D} f {v_R}^{-1} M_{\nu^D}$; (Type II seesaw)
- ➢ Suppose the first term dominates $\longrightarrow$ (A)
  $\mathcal{M}_\nu \simeq \frac{v_{wk}^2}{4v_R{<}\Delta_d{>}}(M_d - M_\ell) \equiv c(M_d - M_\ell)$
  $c \ \sim \ 10^{-10}$
- ➢ (B): $M_\ell = c_u M_u + c_d M_d$;This means $U_\ell \sim 1 + O(\lambda)$ and U_ν determines U_{PMNS} !!.

What is U_ν

☞ $b-\tau$ **mass convergence and large** θ_A

➢ $\mathcal{M}_\nu \;=\; c(M_d - M_\ell);$

But $\mathcal{M}_{\ell,d} = m_{\tau,b}\begin{pmatrix} d\epsilon^4 & a\epsilon^3 & b\epsilon^3 \\ a\epsilon^3 & \epsilon^2 & \epsilon^2 \\ b\epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$ where $\epsilon \sim \lambda \simeq 0.22$;

➢ $$\mathcal{M}_\nu \;=\; m_b\begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & (1 - m_\tau/m_b) \end{pmatrix}$$

➢ So the question now is what is $\frac{m_\tau}{m_b}$ at GUT scale ?

$b - \tau$ masses change with scale

☞ **At M_Z, $m_b \sim 1.7 m_\tau$;**
but all Coupling constants run with energy- so this ratio changes.
define $Y_a = \frac{h_a^2}{4\pi}$**;** $t = ln\mu$
Then $\frac{dlnY_b}{dt} = 6Y_b + Y_t - \frac{7}{15}\alpha_1 - \frac{16}{3}\alpha_3 - 3\alpha_2$

$$\frac{dlnY_\tau}{dt} = 4Y_\tau - \frac{9}{15}\alpha_1 - 3\alpha_2$$

Since $\frac{16}{3}\alpha_3$ **dominates, it pulls b-quark mass down very fast at high energies;**
$\frac{m_b}{m_\tau}(M_U) = \frac{m_b}{m_\tau}(M_Z)[2.5A_t^{-1/2}]^{-1}$**; for** $h_t \simeq 1$**,** $A_t^{-1/2} \sim 0.8$**;**

So m_b gets closer to m_τ and in supersymmetry, there is a parameter $\tan\beta$ and as this changes, the b_τ unification works even better e.g. $\tan\beta \sim 10$.

$b-\tau$ mass convergence and large neutrino mixing

☞ **For** $m_b \sim m_\tau(1+O(\lambda^2))$

➢ $$\mathcal{M}_\nu = m_b c\begin{pmatrix}\lambda^4 & \lambda^4 & \lambda^3\\ \lambda^4 & \lambda^2 & \lambda^2\\ \lambda^3 & \lambda^2 & \lambda^2\end{pmatrix} = m_b c\lambda^2\begin{pmatrix}\lambda^2 & \lambda^2 & \lambda\\ \lambda^2 & 1+\lambda & 1\\ \lambda & 1 & 1\end{pmatrix}$$

➢ Previous discussion of mass matrix for normal hierarchy $\rightarrow$ both atmospheric and solar mixings large

➢ Furthermore, $\frac{\Delta m^2_\odot}{\Delta m^2_A} \sim \lambda^2 \gg (m_\mu/m_\tau)^2$ as required by data $\theta_{13} \simeq \frac{V_{ub}}{1-m_\tau/m_b} \simeq \lambda$

(close to the present upper limit)

Other related recent papers with 126 Higgs

Fukuyama and Okada, (2002); B. Bajc,Alejandra Melfo, Goran Senjanovic, Francesco Vissani, hep-ph/0402122;
H. S. Goh, R. N. M., S. Nasri and S. P. Ng, PLB(2004);
T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada,arXiv:hep-ph/0401213;
C. S. Aulakh and A. Giridhar, hep-ph/0204097;Goh, RNM, Nasri, 2004
S. Bertolini, M. Frigerio and M. Malinsky, arXiv:hep-ph/0406117;
Wei-Min Yang and Zhi-Gang Wang, hep-ph/0406221
Dutta, Mimura, RNM, hep-ph/0406262;
: Case (i) Goh, RNM, Ng; Babu, Macesanu; Bertolini and Malinsky, Schwetz;

SO(10) with extra symmetries: Chen, Mahanthappa (2000); Medeiro Verzilas, King and Ross (2006);..